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FINANCIAL GUARANTEE AS INNOVATION TOOL IN ISLAMIC PROJECT FINANCE

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Abstract

This paper proposes a model to study the arrangement of Islamic project finance with the participation of the government as provider of loan guarantees. The entrepreneur (musharakah) initiates a project and raises funds by issuing Islamic profit sharing debt instruments (mudarabah). The government intervenes in providing financial guarantees in order to enhance the creditworthiness and increase the debt capacity of the project. Our work raises several policy implications related to the structuring of Islamic project finance and the participation of both government and multilateral public agencies such as the Islamic Development Bank. It provides a unifying framework for the improvement of access to funds for Islamic projects and gives a rationale for the government intervention in the arrangement of those projects.
I. INTRODUCTION

The foundations of Islamic finance are described in the Muslim Holy book and the traditions of the Prophet Muhammad (SAW). Under Islamic law (Sharia), making money from money, such as charging interest, is usury and therefore not permitted. Wealth should be generated only through legitimate trade and investment in assets. All forms of interests are forbidden. Moreover, investment in companies involved with illicit activities or goods such as alcohol, gambling, tobacco and pornography is strictly off limits. Islamic financing contracts should be designed to avoid risk-free return and money from money (riba), uncertainty (gharar) and gambling (maysir), e.g., Ebrahim (1999), Esty (2004) and CFA Magazine (2005). Islamic financial instruments have to be carefully structured so that the exchange involves goods for money or partnership shares for money over time. The Islamic financial model works on the basis of risk sharing. Islamic banking is growing at a rate of 15% to 20% per year, e.g., Esty (2004), KPMG Tax Monitor (2005). Some western countries are changing their tax codes in order to accommodate Islamic finance (see KPMG Tax Monitor (2005), BBC News (2004)).¹ The singularities of Islamic finance require the conventional finance approach to be redesigned in order to satisfy the criteria of Islamic financing. These restrictions impose financing constraints to entrepreneurs seeking funds to undertake large scale Islamic project financed investments.

Indeed, project finance is an arrangement in which a sponsor creates a new project company and looks to the project future cash flows as the main source of repayment to lenders. It allows better risk sharing since lenders have to evaluate and audit only the project assets rather than having to assess both project and sponsor assets as would be the case with other financing vehicles. Project finance is an increasingly important method of financing large-scale capital-intensive projects, such as power plants, oil pipelines, automated steel mills, roads, ports, tunnels, etc. The demand for financing often exceeds the supply capacity of the project sponsor and of local capital markets (Farrell, 2003). According to Esty (2004), project-financed investments have grown at a compound rate of almost 20 percent over the past 10 years and globally firms financed 234 billion dollar US of capital expenditures using project finance in 2004, up from 172 billion dollar US in 2003. Kleimeier and Megginson (2001)

¹ According to August 2005 KPMG Tax Monitor: “The last decade has seen tremendous growth in the Islamic financial system. More than 240 financial institutions – in more than 48 countries – now practice some form of Islamic finance. Moreover, Islamic banking assets worldwide are estimated at over US$200 billion with an average annual growth rate of 15 percent in recent years. The UK have published changes to tax rules in an effort to accommodate Islamic financial products. Should Ireland do the same?” For BBC News on Islamic banking go to http://news.bbc.co.uk/2/hi/business/3676138.stm.
compare empirically portfolios of project finance loans to comparable samples of non-project finance loans. They find that project finance loans have longer maturities and are more likely to have third party guarantees. Moreover, projects funded with project finance loans are highly leveraged with an average loan to project value ratio of 67 percent. Ebrahim (1999) establishes a comparative study between Islamic and conventional project finance. Khan (2002) analyses cases involving Islamic instruments in financing build operate and transfer (BOT) Projects.

This paper studies the dynamics between equityholders (musharakah), profit sharing lenders (mudarabah) and the government in the arrangement of Islamic project financed investments. The shareholders (musharakah) being the entrepreneur initiates a project and seeks outside funds to finance it. The entrepreneur finances part of the project investment with own capital and the remaining amount is provided by debtholders in a profit sharing contract (mudarabah). In this financing agreement, lenders share the project after tax net-income with equity holders if the project is successful, but looses their investment in case of default by the project. To reduce the default risk and enhance the creditworthiness of the project, the government intervenes by providing partial financial loan guarantees. If the project turns out to be successful, the government gains tax revenues, debtholders and equityholders share the after tax net-income using a preset sharing formula. We argue that by appropriate risk sharing and/or government financial loan guarantee, project sponsors in the Islamic world can enhance the creditworthiness and increase the debt capacity of their project financed investments.

Since project financed investments involve huge amounts of financing and are highly levered, one way for lenders to hedge credit risk is to require financial guarantees for the loans they make. A financial guarantee is a promise from a third party to make good on payments to the fund provider when the borrower defaults. To have access to more funds and at lower costs, firms resort to financial guarantees to improve their credit rating and debt capacity (The World Bank; 1995, 2002). Government agencies and international organizations such as the World Bank, and Export Credit Agencies are some of the main providers of financial guarantees, especially to back large-scale projects financing (e.g., Dailami and Leipziger (1998), Ehrhardt and Irwin (2004)).

We show that there is a trade-off between the profit sharing and the percentage of loan to be guaranteed. For given levels of debt financing, the entrepreneur has to decide ex-ante whether to give up more profit or seek government guarantee. On one hand, if there is no flexibility over the investment amount, increasing the share of the profit to lenders will result in decreasing equityholders’ net-wealth. On the other hand, if the firm has flexibility over the amount it can invest, increasing either the share of the profit to lenders or the guarantee percentage of the loan will increase the debt capacity of the firm. This increase in the debt capacity consequently induces more investment, thus more taxable income available, which is beneficial for the government and debtholders. Nonetheless, contrary to what we would expect, increasing the portion of the profit share to lenders does not alter the net wealth to the entrepreneur as long as he has the flexibility over the amount to be invested.

The remainder of the paper is as follows. Section II reviews the background of Islamic finance. Section III describes the structure of Islamic project financing. Section IV presents the model and analyses the payoffs of the participants. Section V presents and analyses the numerical simulations results. Section VI concludes.

II. BACKGROUND ON ISLAMIC FINANCE

The foundations of Islamic finance are described in the Muslim Holy book (Quran) and the traditions (Sunnah) of the Prophet Muhammad (S.A.W). Under Islamic law (Sharia), making money from money, such as charging interest, is usury and therefore not permitted. Wealth should be generated only through legitimate trade and investment in assets. All forms of interest are forbidden. Moreover, investment in companies involved with illicit activities or goods such as alcohol, gambling, tobacco and pornography is strictly off limits.

Islamic financing contracts should be designed to avoid risk-free return and money from money ((riba), uncertainty (gharar) and gambling (maysir). They have to be carefully structured so that the exchange involves goods for money or partnership shares for money over time. The Islamic financial model works on the basis of risk sharing. The main financial instruments used in Islamic finance are:

- **Qardh Hasan** is a risk-free asset with nominal rate equal to the inflation rate. Therefore the real rate of return is zero.
- **Murabahah** (cost plus financing and deferred instalment sale) is a form of credit facility which enables customers to purchase equipments/goods without having to take out an
interest bearing loan. The bank buys an item and then sells it on to the customer on a deferred basis.

- **Ijara (operating lease)** is a leasing agreement whereby the bank buys an equipment or productive asset for a customer and then leases it back over a specific period. The client avoids initial capital outlay. In some cases, the customer is able to buy the item at the end of the contract.

- **Mudarabah** is a profit-and-loss sharing Islamic income or revenue bond contract. It offers specialist investment in which the project owner(s) and the investor share any profits. It does not guarantee any fixed rate of return (ribawi), instead, the investor receives a share of the profit or bears the losses generated by the business venture, and the principal is paid (in real terms) at the termination of the contract.

- **Musharakah** is an investment partnership in which profit sharing terms are agreed in advance, and losses are pegged to the amount invested. It is an equity participation or stock ownership contract.

Next, we describe the structure of the Islamic project finance and discuss how the financial instruments mudarabah and musharakah can be used to finance it.

**III. STRUCTURE OF THE ISLAMIC PROJECT FINANCING**

We consider an entrepreneur (or sponsor) who wants to undertake a new investment project. Several owners can co-exist in this new venture; however, we use the term entrepreneur to design the ‘representative equityholder’. The project will be built as a stand-alone firm, meaning that the project is an independent and separate entity. The project requires an initial investment \( I \). The entrepreneur has limited liability; his only commitment in the project is its capital infusion \( S \). Debtholders provide the remaining amount \( D = I - S \). However, the entrepreneur is facing the constraint that the project can be undertaken only if the investment level \( I \) is superior to the minimum required investment \( \bar{I}, I \geq \bar{I} \). In this kind of Islamic financing, the equityholder (musharakah) has to share the profit with debtholders (mudarabah) if the project is successful. In distress situation, the priority goes to debtholders. Debtholders receive fraction of the firm’s after tax net profit plus their principal amount (the principal is paid in real terms not in nominal) if the project is successful. Hence, the owner-entrepreneur keeps the residual value of the project.
If the project is undertaken, the government benefits from it because of the corporate taxes. Therefore, it is in the interest of the government to encourage the entrepreneur to go ahead with the project. But, in the Islamic context, since the conventional capital market channel cannot always be used, it can be hard for the entrepreneur to raise enough money in order to finance the project. To overcome that, the government will intervene by providing partial loan guarantees for the project loan. There are many ways for the government to do so. One way will be through multilateral public agencies such as the Islamic Development Bank. The government will act so as far as the net social benefit outweighs the loan guarantee cost. The maximum government guarantee coverage should be obtained when the maximum benefit is equal to the marginal cost of guaranteeing the loan. We use as proxy for the social benefit, the amount of tax the government receives from the project.

Figure 1 describes the relationships between the entrepreneur (shareholders), lenders and the government. The project is financed by the entrepreneur and debtholders. The government enhances the creditworthiness of the project by providing loan guarantees. If the total financing exceeds the minimum capital requirement, the project goes ahead, otherwise the project is abandoned. When the project goes ahead, the investment is made and the project yields cash revenues. The cash revenues are then redistributed among the players, i.e., the government collects taxes, debtholders receive their repayment which is composed of the principal and share of the profit, and equity holders are left with the residual value. Therefore, the success of the project benefits all parties involved which is consistent with the risk sharing philosophy behind Islamic finance.

Based on this Islamic project finance structure, next we present the model and analyse the payoffs to the participants.

IV. THE MODEL

As described in the previous section, we consider an entrepreneur (or sponsor) who wants to undertake a new investment project. The project will be built as a stand-alone firm, meaning that the project is an independent and separate entity. The project requires an initial investment $I$ and its cash flows are characterized by the following technology $V = \tilde{\theta}v(I)$ where $\tilde{\theta}$ is the random output price which captures the stochastic nature of the cash flows and $v(I)$ is a twice differentiable concave function of investment. We assume the project cash flows to
have the following risk adjusted stochastic process:

\[ dV_t(I) = (\mu - \lambda \sigma)V_t(I)dt + \sigma V_t(I)dW_t, \]  

(1)

where \( \mu, \sigma \) and \( \lambda \) are respectively the instantaneous return, volatility and market price of risk of the project’s asset returns. The random variable \( W \) is a standard Wiener process.

In Islamic finance, there is no risk-free rate, but instead any money deposit in a bank account and not used for investment will grow at the inflation rate \( \pi \). Under the risk neutral valuation à la Harrison and Kreps (1979) and Merton (1974), the parameter \( \lambda \) is the market price of risk for the project. In the special case of a tradable security, \( \mu - \lambda \sigma \) is equal to the inflation rate \( \pi \) in equation (1).³

As mentioned before, the project requires an initial endogenous investment level \( I \), but there is a minimum investment requirement in the sense that the entrepreneur has to raise at least \( \tilde{I} \) before the project can be undertaken. The project sponsor (or shareholders) has limited financing capacity, therefore he can only finance an amount \( S \) of the total investment. He then needs outside financing in the form of profit and loss sharing certificate. In other words, the entrepreneur can infuse a capital level \( S \) and has to borrow \( I - S \) to finance the new project. We assume a simple capital structure for the project, consisting of single debt and equity contracts. The maturity of the project is \( T \). We assume the existence of corporate taxes. The project is owned by the entrepreneur and the project cash flows are used to pay the project debtholders. In this financing framework, often referred to as non- or limited recourse financing, lenders depend on the performance of the project itself for repayment rather than the credit of the sponsor. The only commitment of the sponsor is its capital contribution.

In the following subsections, we present respectively, the cost/benefit to the government, the payoff from the participation constraint of lenders, and the payoff to the owner-entrepreneur.

**IV.1. Guarantee cost and tax revenue to the government**

The government acts as a stakeholder in the project by providing partial loan guarantees. The incentive for the government is to gain positive social benefits if the project is realized. The government insures a fraction \( \omega \) of the loan principal, with \( 0 \leq \omega \leq I \). From the structural approach of Merton (1974, 1977), the total loan guarantee cost to the government is

equivalent to a put option and is given as follows

\[
G = E\left[e^{-\pi T} \max(\omega D - V_I(I), 0)\right] = \omega e^{-\pi T} DN(-z + \sigma \sqrt{T}) - V_0(I) e^{(\mu - \lambda \sigma - \pi)T} N(-z),
\]

where \( z = \frac{\ln(V_0(I)/\omega D) + (\mu - \lambda \sigma + 0.5\sigma^2)T}{\sigma \sqrt{T}} \) and \( N(.) \) is the cumulative normal distribution function.

By guaranteeing the debt, the government expects to raise more taxes to fulfill its social agenda. If the project is realized, the expected tax revenue to the government will be

\[
Taxes = E\left[e^{-\pi T} \tau_c \max(V_I(I) - I, 0)\right] = \tau_c V_0(I) e^{(\mu - \lambda \sigma - \pi)T} N(x) - e^{-\pi T} \tau_c IN(x - \sigma \sqrt{T}),
\]

where \( x = \frac{\ln(V_0(I)/I) + (\mu - \lambda \sigma + 0.5\sigma^2)T}{\sigma \sqrt{T}} \) and \( \tau_c \) is the corporate tax rate.

The net gain to the government is the difference between the tax gain and the cost of the guarantee, i.e., \( Taxes - G \). Therefore, the participation constraint of the government is such that the net gain to him is positive, \( Taxes - G \geq 0 \).

**IV.2. Lenders’ participation constraint and project debt capacity**

As we mentioned above, the project is financed with equity and debt. The entrepreneur contributes \( S \) and the rest \( I - S \) is financed by outside investors in the form of profit-sharing Islamic bond instrument. Outside investors share the profit with the owner-entrepreneur. At the maturity \( T \), if the project is successful, debtholders receive their principal otherwise they lose money since they won’t be able to recover the full amount of the principal.

- **Optimal debt without financial guarantee**

In absence of loan guarantee, the payments to debtholders at date \( T \) are:

\[
D_T = \begin{cases} 
D + q(1 - \tau_c)(V_I(I) - I) & \text{if } V_I(I) > I \\
D & \text{if } D < V_I(I) \leq I, \\
V_I(I) & \text{if } V_I(I) \leq D
\end{cases}
\]

where \( q \) is the profit sharing allocation parameter.

Equation (4) states that if the cash flow generated by the project exceeds the initial investment amount, debtholders receive their principal amount \( D \) and share of the after tax net-income (percentage \( q \)). Otherwise, they receive only the principal amount when the project end value is greater than the principal amount and the project savage value in case the project
market value is inferior to the principal amount. Therefore, the equilibrium value of the debt today is the expected present value of the terminal payments to debt holders:

\[
D = E[e^{-\alpha T} q(1-\tau_c) \max(V_r(I)-L,0)] + E[e^{-\alpha T} D \times 1_{[V_r(I)]>D}] + E[e^{-\alpha T} V_r(I) \times 1_{[V_r(I)]<D}]
\]

\[
= q(1-\tau_c) \left[V_0(I)e^{(\mu-\lambda \sigma-\gamma)T} N(x) - e^{-\alpha T} IN(x-\sigma\sqrt{T}) \right]
\]

\[
+ e^{-\alpha T} DN(y-\sigma\sqrt{T}) + V_0(I)e^{(\mu-\lambda \sigma-\gamma)T} N(-y)
\]

or

\[
D = V_0(I)e^{(\mu-\lambda \sigma-\gamma)T} q(1-\tau_c) N(x) + N(-y)
\]

\[
- q(1-\tau_c)e^{-\alpha T} IN(x-\sigma\sqrt{T}) + e^{-\alpha T} DN(y-\sigma\sqrt{T})
\]

where \( x = \frac{\ln(V_0(I)/I) + (\mu - \lambda \sigma + 0.5 \sigma^2)T}{\sigma\sqrt{T}} \) and \( y = \frac{\ln(V_0(I)/D) + (\mu - \lambda \sigma + 0.5 \sigma^2)T}{\sigma\sqrt{T}} \).

After few algebraic manipulations, we can show that the maximum total amount of debt without guarantee will be

\[
D = V_0(I)e^{(\mu-\lambda \sigma-\gamma)T} q(1-\tau_c) N(x) + N(-y) - q(1-\tau_c)e^{-\alpha T} IN(x-\sigma\sqrt{T})
\]

\[
1 - e^{-\alpha T} N(y-\sigma\sqrt{T})
\]

Given the level of sponsor contribution \( S \) or total investment \( I \) to be made, equation (6) allows us to solve for the maximum level of debt outside investors will be willing to extend to the project. Equation (6) is a fixed point problem as \( x \) and \( y \) contains \( D \), we solve for the level of maximum debt using Matlab optimization toolbox. We denote by \( D_{NG} \) the optimal debt level without financial guarantee. If the entrepreneur were to change the investment level, the debt amount will also change.

- **Optimal debt with financial guarantee**

  Instead, in the presence of financial guarantee, the value of the guaranteed debt will be equal to the value of the debt without guarantee plus the value of the guarantee, i.e., \( D = D_{NG} + G \). Using equations (2) and (4), we obtain

\[
D = E[e^{-\alpha T} D_r] + G
\]

\[
= V_0(I)e^{(\mu-\lambda \sigma-\gamma)T} q(1-\tau_c) N(x) + N(-y) - N(-z) - q(1-\tau_c)e^{-\alpha T} IN(x-\sigma\sqrt{T})
\]

\[
+ e^{-\alpha T} \left[N(y-\sigma\sqrt{T}) + \alpha N(-z + \sigma\sqrt{T}) \right]
\]

which yields,
\[ D = \frac{V_0(I)e^{(\sigma - \frac{1}{2}\sigma^2)T}}{1 - e^{-\sigma T}N(y - \sigma \sqrt{T}) - e^{-\sigma T}N(z + \sigma \sqrt{T})} \left[ q(1 - \tau_c)N(x) + N(-y) - N(-z) \right] - q(1 - \tau_c)e^{-\sigma T}IN(x - \sigma \sqrt{T}) \] \quad (8)

Given the level of sponsor contribution \( S \) or investment \( I \), equation (8) is a fixed point problem as \( x, y \) and \( z \) contains \( D \). We use Matlab optimization toolbox to solve for \( D \). We denote by \( D_G \) the optimal maximum guaranteed debt. In conventional modern finance, in presence of credit insurance, debtholders will either decrease the credit premium they require to lend to the firm or they will increase their funding to the firm because the credit insurance enhances the project creditworthiness and increases its borrowing capacity. In Islamic banking, there is no return premium; the credit improvement will translate through changes in the profit sharing parameter \( q \) and/or the debt capacity \( D_G \).

Figure 2 plots the payoffs to lenders as function of the value of the project. In the first graph (top graph), the payoff of the non-insured debt is plotted. As illustrated, in bad performance states, lenders loose their initial investment, but in good states, they recover the principal of their investment plus share of the profit. In the second graph (bottom graph), the payoff to the insured lenders is plotted. As illustrated by the graph, the government loan guarantee reduces the downside losses to lenders. In good performance states, lenders still receive their principal and share of the profit.

Insert Figure 2 here

**IV.3. Net-wealth to the entrepreneur-owner**

The project borrows \( D \) at the beginning and promise to pay fraction \( q \) of the after tax net-income plus the principal amount \( D \) at date \( T \). Hence, the owner-entrepreneur keeps the residual value of the project. Therefore, the residual to the owner-entrepreneur in the new venture at time \( T \) is as follows:

\[
R_T = \begin{cases} 
V_T(I) - D - q(1 - \tau_c)(V_T(I) - I) - \tau_c(V_T(I) - I) & \text{if } V_T(I) > I \\
V_T(I) - D & \text{if } D < V_T(I) \leq I, \\
0 & \text{if } V_T(I) \leq D
\end{cases}
\] \quad (9)

Equation (9) states that the residual value to the entrepreneur is the difference between the project value and the payments to debtholders and government when the project value is greater than the initial investment amount. Otherwise, it is either the difference between the firm value and the principal payment or zero whichever is greater. The net-wealth of the
owner-entrepreneur in the new venture is the expected present value of its terminal payoff given in equation (9) minus his initial capital contribution:

\[ R = E[e^{-\pi T} R_T] - S, \]  

or

\[ R = V_0(I)e^{(\mu - \lambda \sigma^2)T} N(y) - e^{-\pi T}DN(y - \sigma \sqrt{T}) \]
\[ - (q(1 - \tau_e) + \tau_e) \left [ V_0(I)e^{(\mu - \lambda \sigma^2)T} N(x) - e^{-\pi T}IN(x - \sigma \sqrt{T}) \right ] - (I - D). \]  

From equation (10), the owner-entrepreneur is the residual claimant. The residual claim represents the value of the firm minus the expected value to debtholders and tax payment to the government. The guarantee cost is a deadweight cost for the economy; however, at the government level, it is compensated by the tax gain.

Figure 3 plots the payoff to the owner-entrepreneur as function of the project value. As illustrated by the graph, if the project turns out to be unsuccessful, the owner-entrepreneur receives nothing and loses only his initial capital investment since he has limited liabilities. But if the project succeeds the owner-entrepreneur receives the residual value net of his initial capital investment.

\[ \text{Insert Figure 3 here} \]

As stated previously, the owner-entrepreneur has the flexibility over the choice of the level of investment \( I \) up to the maximum available funds. He can decide whether to go ahead with the project or not. Therefore, he will undertake the project only if the expected net gains to him are positive. All else being equal, the optimal level of \( I \) from the viewpoint of the entrepreneur will be the one maximizing his net wealth \( R \).

Next we perform several numerical analyses with the optimization toolbox of Matlab using defined sets of parameters values to gauge the impact of the sharing rule and the government guarantee on the optimal policies.

V. SIMULATIONS RESULTS AND POLICY IMPLICATIONS

Several numerical analyses will be conducted to gauge for the effects of the changes in key policy parameters values by keeping others constant. Several parameters values such as the volatility level, the maturity of the project, etc., are set based on empirical evidence by Ebrahim (1999), Kleimeier and Megginson (2001), Khan (2002) and Esty (2004).
V.1. The role of financial guarantee in absence of investment flexibility

We define the absence of investment flexibility by the fact that the project requires an exact investment amount and there is no room for the entrepreneur to change the level of the investment, in other worlds, either he raises the required amount or the project is not undertaken. The numerical simulation results are presented in Figure 4.

**Insert Figure 4 here**

In Figure 4, without loss of generality, the level of investment $I$ is normalized to 100. We analyze the sensibilities of the optimal policies (the government guarantee percentage, the cost of the guarantee to the government, the entrepreneur’s net wealth and the debt ratio) to the profit sharing parameter $q$ for different levels of capital contribution from the entrepreneur measured as the percentage of sponsor own capital in total investment. From the percentage capital level of the entrepreneur, we compute the amount of the capital contribution by the entrepreneur $S$. The debt amount to be raised is therefore the difference between the investment and the entrepreneur capital, i.e., $I - S$. To raise the required debt level, the entrepreneur needs financial guarantees as illustrated in Figure 4.

We observe that the government loan guarantee enhances the creditworthiness of the project. However, the firm needs less government guarantees when the profit sharing parameter is high and the sponsor’s own capital is big. Therefore, with limited capital, the entrepreneur trades-off between the fraction of the profit to extend to lenders and the partial guarantee portion. When the profit sharing parameter increases, the partial guarantee portion decreases. In this non-flexible investment environment, increasing the profit sharing parameter $q$ decreases the net-wealth to the entrepreneur. Intuitively, since the investment level does not change, the cash flow level is the same, and so is the before tax net-income. As a consequence, the government net-wealth is decreasing with the cost of the guarantee. We should keep in mind that we have assumed no investment flexibility; in presence of investment flexibility, the outcome could be different.

V.2. The role of financial guarantees with investment flexibility

In contrast to the absence of investment flexibility, here we assume that the entrepreneur can increase or decrease the level of the investment as long as the investment
level is above the minimum required investment. For the numerical simulations, we assume the capital contribution from the entrepreneur to be fixed, but the investment level is varying with the debt capacity which is function of the percentage of guarantee provided and the sharing rule parameter. The numerical simulations results are plotted in Figure 5.

**Insert Figure 5 here**

The graphs show the simultaneous impacts of the government loan guarantee and the profit sharing parameter on the optimal policies (the maximum debt capacity, the investment level, the entrepreneur’s net-wealth, the guarantee cost, the total tax revenue and the government net-wealth). The entrepreneur capital contribution $S$ is fixed (without loss of generality, we normalized it to 100) and the total investment is equal to the entrepreneur contributed capital $S$ plus the total debt $D$, $I = S + D$. The flexibility over the investment level comes from the amount of debt raised. If lenders are willing to extend more funds to the project, then the project investment will be high otherwise it will be low.

As illustrated by the graphs, the debt capacity ($D$) of the project increases when either the government percentage guarantee increases or the profit sharing parameter increases. In this model, the investment level is endogenous since any changes on the debt level affect the total investment level. On one hand, we observe that, increasing the profit sharing parameter $q$ does increase the entrepreneur’s net-wealth. Indeed, since the capital contribution of the entrepreneur is fixed, increasing the profit sharing parameter increases the debt capacity, which in turn increases the total investment level inducing more available profit to be shared. Therefore, the increasing size effect dominates the decreasing sharing fraction.

On the other hand, increasing the loan guarantee percentage improves the entrepreneur and government net-wealth. The explanation is as follows. With more guarantees, the firm is able to borrow more, yielding more investment, which increases the taxable income and the residual value to shareholders. For the government, more taxable income generates more tax revenues, and with our set of parameters values used to generate the graphs, the tax gain outweighs the guarantee cost for the government.

V.3. **Financial guarantees and analysis of the costs/benefits to the government**

As illustrated by the government net wealth graph in Figure 5, providing more guarantees always improves the net wealth to the entrepreneur. But for the government, it is not always beneficial. The guarantee portion or the profit sharing parameter increases the debt
capacity of the project which increases the total investment since \( I = S + D \). Using the government loan guarantee to enhance the creditworthiness and increase the debt capacity of the project induces a cost for the government but at the same time, if the project is successful, the government gains from tax revenues collected. Therefore, there is an optimal guarantee level to be provided by the government. The optimal guarantee portion is obtained when the marginal gain (marginal tax revenue) is equal to the marginal cost (marginal guarantee cost).

Insert Figure 5 here

V.4. Sensitivity analyses to the parameters values

- **Sensitivity to the output price**

  Figure 6 presents the impact of the simultaneous changes in the loan guarantee percentage and the profit sharing parameter on the optimal policies (the debt capacity, the net-wealth to the entrepreneur and the government net-wealth) for two levels of output price. For lower output prices, it takes more guarantees to insure a positive net-wealth to the entrepreneur. Or put in other words, it takes more guarantees for the project to go ahead. But for the government, with lower levels of output price, it is more beneficial to lower the percentage of guarantee it provides if the project goes ahead. Unfortunately, that will not be the case if the entrepreneur does not expect positive residual value. The government gets nothing when the project is abandoned; therefore, it is in its interest to provide some level of financial guarantee which will create incentive for the entrepreneur (positive net-wealth) to go ahead with the project. Hence, even though the zero guarantee contribution seems more plausible for the government at first look; it does not guarantee that the project will go ahead.

Insert Figure 6 here

- **Sensitivity to the project risk level**

  Figure 7 illustrates the sensitivity results with respect to the project’s asset returns volatility. When the project is too risky, debtholders require more part of the profit for compensation for their investment which decreases the net-wealth of the entrepreneur. Therefore, more needs for guarantees, which affects negatively the net-wealth of the government. As we have argued previously, it is in the interest of the government to provide financial guarantee support even if it results in destruction of part of its net-wealth, otherwise the project will not go ahead. The government will act so as long as its net-wealth remain
positive.

**Insert Figure 7 here**

- Sensitivity to the project’s asset growth rate, the market price of risk and the inflation rate

Figures 8, 9 and 10 show the sensitivity results respectively for changes in the project asset returns growth, the market price of risk and the inflation rate. Lower levels of project asset growth rate (or higher levels of either the market price of risk or the inflation rate), all else being equal, imply less future expect value for sharing between the three players (debtholders, entrepreneur and government).

**Insert Figure 8 here**

**Insert Figure 9 here**

**Insert Figure 10 here**

- Sensitivity to the project maturity

Figure 11 shows the sensitivity results with respect to the project maturity. When the maturity of the project increases, the net-wealth to the government and the entrepreneur decrease, and that because the amount of debt extended to the project is lower. The intuition is as follows. Recall, from equations (4) and (7), the face value of the debt is the initial amount of debt since no money is made from nothing in Islamic financing, therefore the discounting factor has more impact with longer maturities than shorter maturities. As a consequence, debtholders require more share of the profit. However, increasing the profit sharing parameter will create less incentive for the entrepreneur to invest.

**Insert Figure 11 here**

- Sensitivity to the tax rate

Figure 12 exhibits the sensitivity results with respect to changes in the tax rate. When the tax rate increases, it creates less incentive for investment, therefore the total net taxable income is lower, which brings less revenue to the government. This phenomenon is more accentuated for higher levels of the profit sharing parameter. Hence, for the government, increasing the tax rate will create the inverse effect, which is the decrease in the government revenue since it does not create incentive for investment.

**Insert Figure 12 here**

- Sensitivity to the entrepreneur’s capital contribution
Figure 13 plots the sensitivity results with respect to level of the entrepreneur’s capital contribution. For these graphs, we take two values for the contribution of the entrepreneur, \( S = 50 \) and \( S = 100 \). We observe that when the value of \( S \) doubles, all the optimal policy values are doubled. Therefore, the optimal policies are proportional to the value of \( S \). This is true because the revenue function used in our simulation is linear to the investment level. With non-linear revenue function specification, this proportionality won’t hold. However, all the qualitative results obtained above will remain valid.

**Insert Figure 13 here**

**VI. CONCLUSION**

This paper proposes a model to study the interactions between the project’s sponsor, its lenders and the government. The entrepreneur has an investment opportunity with positive expected net gain. To finance the project, the entrepreneur contributes with own capital and seeks outside funds by issuing Islamic profit sharing certificates. He also requires government financial guarantees to enhance the creditworthiness and increase the debt capacity of the project. The payoffs to all the participants are derived from their participation and incentive constraints in equilibrium. The participation constraint of debtholders determines the optimal profit sharing parameter and the maximum debt amount. The percentage of loan guarantee is determined such that the government gains positive net-wealth from the project. The investment decision lies on the entrepreneur and he gets the residual value of the project.

Our work raises several policy implications related to the arrangement of Islamic project finance and the participation of both government and multilateral public agencies such as the Islamic Development Bank. It provides a unifying framework for the improvement of access to funds in the Islamic financing context and gives a rationale for the government intervention in the arrangement of Islamic project finance.
REFERENCES


Figure 1: The players chart flow

This chart plots the interactions between the entrepreneur, lenders and the government in the arrangement of Islamic project finance. The entrepreneur initiates a project idea and requires outside financing in the form of Islamic profit sharing debt. The government intervene by providing a financial guarantee in order to improve the project creditworthiness. If the investment is made, each stakeholder receives part of the cash flows generated by the project. The chart illustrates the cash inflows to and outflows from the project to the different stakeholders.
Figure 2: Payoff to lenders (madarabah)

This graph represents the payoff to a holder of an Islamic profit sharing revenue bond. This bond consists of a security which pays a fraction of the project profit and the principal to the holder if the project is successful, otherwise the security holder receive the salvage value of the project in case of default. The top graph represents the payoff to non-insured lenders, while the bottom graph represents the payoff to insured lenders. Comparing the two graphs, we observe that, the financial guarantee limits the downside losses, while the security holder still maintains the option to profit any potential gain. The graphs have been constructed using the following parameters values: $q = 0.20$, $\omega = 0.50$, $\tau_c = 0.35$, $D = 60$, $I = 100$. The payoffs are obtained by subtracting from the final payment to lenders ($D_T$) the principal $D$: $D_T - D$. 

![Payoff to non-insured lenders](image1)

![Payoff to insured lenders](image2)
This graph represents the payoff to the entrepreneur or shareholders. The entrepreneur receives the residual value of the project. If the project is successful, the entrepreneur shares the after tax profit with lenders, otherwise he looses his initial investment. Comparing this payoff to the payoff of standard call option, commonly used to modelled equity, here there is a kink when the payoff crosses the horizontal line. The curve above the horizontal line is the profit sharing part presented in Islamic financing. The graphs have been constructed using the following parameters values: $q = 0.20$, $\omega = 0.50$, $\tau_c = 0.35$, $D = 60$, $I = 100$, $S = I - D = 40$. The payoffs are obtained by subtracting from the final payment to the entrepreneur ($R_T$) his initial capital investment $S$: $R_T - S$. 

![Payoff to shareholders](image.png)
Figure 4: Optimal policies with fixed level of investment

This figure is composed of four graphs plotting the optimal policies as function of the profit sharing fraction \( q \) for different levels of entrepreneur’s capital contribution portion \( S / I \). The graphs plot respectively, (a) the percentage of loan guarantee by the government \( \omega \), (b) the cost of the financial guarantee to the government, (c) the entrepreneur’s net-wealth, and (d) the project debt ratio which is equal to the total debt divided by the sum of the total debt and the total equity. The graphs are computed using the following parameters values: \( V_0(I) = 1.5 \times I \), \( \mu = 0.08 \), \( \sigma = 0.4 \), \( \lambda = 0.1625 \), \( \pi = 0.015 \), \( \tau_c = 0.35 \), \( T = 10 \); the total investment level has been normalized to \( I = 100 \). The capital contribution percentage of the entrepreneur is \( S / I \), which gives the value of \( S \), and the guaranteed debt value is obtained by \( I - S \). The percentage \( \omega \) is the guarantee percentage need to raise the debt amount \( I - S \).

(a) Percentage of loan guaranteed \( (\omega) \)  
(b) Guarantee costs \( (G) \)

(c) Entrepreneur net-wealth \( (R) \)  
(d) Debt ratio \( (D / (R + I)) \)
This figure is composed of six graphs. The graphs plot respectively top to bottom and left to right, the maximum debt capacity of the project, the maximum total investment level of project, the net-wealth to the entrepreneur, the cost of the financial guarantee to the government, the total taxes collected by the government, and the net-wealth of the government. The graphs are computed using the following parameters values: $V_0(I) = 1.5 I$, $\mu = 0.08$, $\sigma = 0.4$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$. 
Figure 6: Sensitivity to the output price

This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the cash flow function defined as $V(I) = 1.2 \times I$. The bottom graphs, panel (b) plot the same variables but for cash flow function $V(I) = 1.5 \times I$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\lambda$ = 0.1625, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

(a) $V(I) = 1.2 \times I$

(b) $V(I) = 1.5 \times I$
Figure 7: Sensitivity to the project risk level

This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the project risk level set at $\sigma = 0.40$. The bottom graphs, panel (b) plot the same variables but for the project risk level set at $\sigma = 0.60$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

(a) $\sigma = 0.40$

(b) $\sigma = 0.60$
Figure 8: Sensitivity to the asset growth rate

This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the project asset’s growth rate set at $\mu = 0.05$. The bottom graphs, panel (b) plot the same variables but for the project asset’s growth rate set at $\mu = 0.08$. The graphs are computed using the following parameters values: $\sigma = 0.40$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

(a) $\mu = 0.05$

(b) $\mu = 0.08$
Figure 9: Sensitivity to the market price of risk

This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the market price of risk set at $\lambda = 0.1625$. The bottom graphs, panel (b) plot the same variables but for the market price of risk set at $\lambda = 0.20$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\pi = 0.015$, $z_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

(a) $\lambda = 0.1625$

(b) $\lambda = 0.20$
Figure 10: Sensitivity to the inflation rate

This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the inflation rate set at \( \pi = 0.015 \). The bottom graphs, panel (b) plot the same variables but for the inflation rate set at \( \pi = 0.03 \). The graphs are computed using the following parameters values: \( \mu = 0.08, \sigma = 0.40, \lambda = 0.1625, \tau_c = 0.35, T = 10 \); the entrepreneur contributed capital has been normalized to \( S = 100 \). The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: \( I = S + D \).

(a) \( \pi = 0.015 \)

(b) \( \pi = 0.030 \)
Figure 11: Sensitivity to the maturity of the project

This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the maturity of the project set at $T = 10$ years. The bottom graphs, panel (b) plot the same variables but for the maturity of the project set at $T = 30$ years. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

(a) $T = 10$ years

(b) $T = 30$ years
Figure 12: Sensitivity to the tax rate

This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the tax rate set at $\tau_c = 0.35$. The bottom graphs, panel (b) plot the same variables but for the tax rate set at $\tau_c = 0.50$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\lambda = 0.1625$, $\pi = 0.015$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

(a) $\tau_c = 0.35$

(b) $\tau_c = 0.50$
Figure 13: Sensitivity to the entrepreneur contributed capital

This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the entrepreneur contributed capital set to $S = 50$. The bottom graphs, panel (b) plot the same variables but for the entrepreneur contributed capital set to $S = 100$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

(a) $S = 50$

(b) $S = 100$